

Algebra 2

9-00 Measures of Center and Dispersion

Measure of central tendency

- A number used to represent the _____ or _____ of a set of data values.

Mean

- _____, of n numbers is the _____ of the numbers divided by _____.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Median

- _____ number when the numbers are written in _____. (If n is even, the median is the _____ of the two _____ numbers.)

Mode

- Number or numbers that occur most _____. There may be _____ mode, _____ mode, or _____ mode.

The winning scores of 6 baseball games are 5, 7, 8, 5, 10, 3. Find the mean, median, and mode.

Measure of dispersion

- Statistic that tells you how _____, or _____, data values are.

Range

- _____ between the _____ and _____ data values.

$$\text{Range} = \text{max} - \text{min}$$

Standard deviation

- Describes the _____ differences (or deviation) between a data's _____ and the _____.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

Find the standard deviation of the following data set.

4,8,12,15,3

Finding the standard deviation on a graphing calculator**TI calculator**

- [STAT] → Edit, Enter data values in L1 (clear list first)
- [STAT] → CALC → 1-Var Stats, [ENTER] x2, Find σ_x

NumWorks

- Select Statistics from home
- In Data tab
- Enter data in Value V1 list
- In Stats tab
- Read standard deviation from list

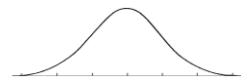
463 #1-2, 464 #1-7 = 9

Algebra 2

9-01 Using Normal Distributions

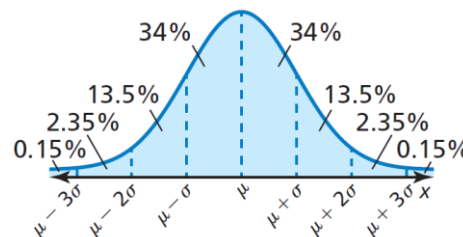
Normal Distribution

- A normal distribution is modeled by a _____-shaped curve called a _____ curve that is symmetric about the _____.



Normal Distribution Properties

- A normal distribution with mean μ and standard deviation σ has the following properties:
 - The total area under the related normal curve is _____.
 - About _____ of the area lies within _____ standard deviation of the mean.
 - About _____ of the area lies within _____ standard deviations of the mean.
 - About _____ of the area lies within _____ standard deviations of the mean.



A normal distribution has mean and standard deviation. For a randomly selected x -value from the distribution, find $P(\mu - \sigma \leq x \leq \mu + 3\sigma)$

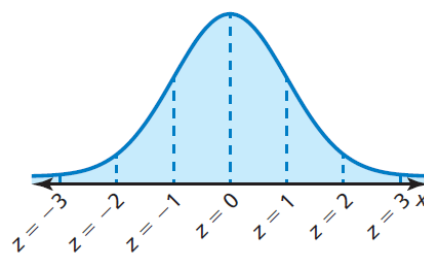
$$P(x \leq \mu - \sigma)$$

The weight of strawberry packages is normally distributed with a mean of 16.18 oz and standard deviation of 0.34 oz. If you randomly choose a container, what is the probability that it weighs less than 15.5 oz?

$$\mu = 33, \sigma = 4, \text{ find } P(29 \leq x \leq 37)$$

Standard Normal Distribution

- Normal distribution with mean = ____ and standard deviation = ____.
- Formula = $z = \frac{x - \mu}{\sigma}$
- The z value for a particular x-value is called the _____ for the x-value and is the number of _____ the x-value lies above or below the _____ \bar{x} .

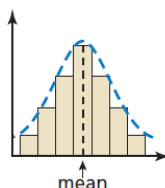


A survey of 20 colleges found that the average credit card debt for seniors was \$3450. The debt was normally distributed with a standard deviation of \$1175. Find the z-score corresponding to an x-value of \$3600.

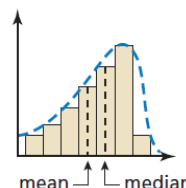
$\sigma = 34$, z-score = -1.5, $x = 138$ what is μ ?

Skewed

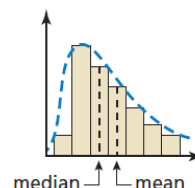
- Normal distribution: mean _____ median
- Skewed distribution: mean _____ median
- If mean < median, skewed _____
- If mean > median, skewed _____



Bell-shaped and

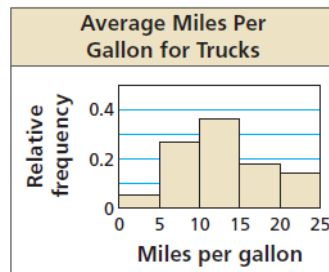
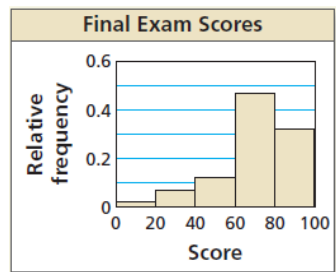


Skewed left



Skewed right

Determine whether each histogram has a normal distribution.



470 #1, 3, 5, 7, 9, 11, 13, 15, 17, 18, 19, 23, 24, 33, 37, 39, 41, 43, 45, 47 = 20

Algebra 2

9-02 Populations, Samples, and Hypotheses

Population

- Collection of _____ data of interest.
- i.e. all the people

Sample

- _____ of the population
- i.e. only the people surveyed

Identify the population and the sample. Describe the sample.

a. The owner of a dance studio surveys 32 dancers and finds that 25 of them prefer hip hop.

b. A counselor at a middle school reviews 225 students' class schedules and finds that 46 students have a science class during first period.

Parameter

- Numerical description of a _____ characteristic

Statistic

- Numerical description of a _____ characteristic

For all teenagers who had jobs last summer in a certain town, the mean hourly wage was \$8.25. Is the mean hourly wage a parameter or a statistic?

A survey of 912 men, ages 50–60 in Central America, found that the standard deviation of the lengths of their feet is about 4 centimeters. Is the standard deviation of the lengths of their feet a parameter or a statistic?

Proportion

- Ratio of _____ with that characteristic to _____ number of members

Use technology to simulate flipping a coin 20 times. What proportion of the 20 flips result in heads?

PRB → randint(1,2)

random.org/coins

random.org/dice

Hypothesis

- Claim about a _____ characteristic
- To analyze, distinguish between results that occur by _____ and those that are _____ to occur by chance
- Using the simulator at andrews.edu/~rwright/algebra2/SamplingSimulator.html, the sample proportion needs to be in the middle _____ of the histogram. If _____ samples are simulated, then reject the hypothesis if it is in the _____ points on either end of the histogram.

You roll a six-sided die 5 times and do not get an even number. The probability of this happening is $\left(\frac{1}{2}\right)^5 = 0.03125$, so you suspect this die favors odd numbers. The die maker claims the die does not favor odd numbers or even numbers. What should you conclude when you roll the actual die 50 times and get (a) 23 odd numbers and (b) 40 odd numbers?

478 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 22, 23, 25, 29, 31, 33, 35, 39, 41 = 20

Algebra 2

9-03 Collecting Data

Steps in a Statistical Study

1. Identify _____ of interest and _____
2. Choose _____ that represents the population
3. Collect _____
4. _____ and _____ the data using a statistic
5. _____ the data, make _____, draw _____ about the population

Types of Samples

- Random
 - Each member _____ likely to be selected
- Self-Selected
 - Members _____
- Systematic
 - A _____ is used to choose members
- Stratified
 - Population divided into groups. Members _____ chosen from _____ group
- Cluster
 - Population divided into groups. Entire _____ are chosen randomly.
- Convenience
 - Members who are _____ to reach

You want to determine whether the people in your neighborhood like the new social media website that provides neighborhood updates. Identify the type of sample described.

a. You ask all the people who live on your block.

b. You randomly select a person from each block in the neighborhood.

c. You email questionnaires and use only the questionnaires that are returned.

d. You divide the people in your neighborhood according to even and odd house numbers, then select all the people with an even house number.

Bias

- _____ that results from a sample that _____ the population
- Unbiased sample
 - Is _____ of the population
- Biased sample
 - _____ - or _____ -represents a part of the population

Identify the type of sample and explain why the sample is biased.

- a. The principal asks students at one lunch table about the quality of food served in the school's cafeteria.
- b. A sports announcer wants to know how often people in the town attend community sporting events. She asks every tenth person who enters the field to watch a local soccer game.

You are a member of your school's banquet committee. You want to poll the members of the senior class to find out where the banquet should be held. There are 75 students in the senior class. Describe a method for selecting a random sample of 40 seniors to poll.

Methods of Data Collection

- Experiment
 - Imposes a _____ on a sample
- Observational Survey
 - _____ the sample
- Survey
 - _____ the sample some _____
- Simulation
 - Using a _____ to reproduce a situation

Identify the method of data collection described in each situation.

A teacher records how many students turn in their homework as they enter the classroom.

A manager uses a computer program to estimate the number of defective products that will be produced by a particular assembly line.

A town supervisor surveys residents of the town by asking, "Should the town build a playground and a dog area in the park on Main Street?" Explain why the question may be biased or otherwise introduce bias into the survey. Then describe a way to correct the flaw.

486 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 39, 41, 43, 49 = 20

Algebra 2

9-04 Experimental Design

Describing Experiments

Controlled Experiment

- Groups have same _____ except for the _____ of interest
- Treatment group: has the _____
- Control group: _____ (sometimes has _____)

Randomized comparative experiment

- Members are _____ assigned to the groups to try to eliminate _____

Determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

At a car dealership, customers were given the choice of whether to have side cameras installed on their new cars for free. Forty car owners who had the cameras installed were monitored for two years, as were forty car owners who did not have side cameras installed. At the end of two years, car owners who had side cameras installed had 22% fewer car accidents than car owners in the other group.

Comparative Studies and Causality

- Experiment
 - Well-designed _____ by eliminating other variables can conclude _____
- Observation
 - Can show _____, but not _____

Determine whether the following research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.

A researcher wants to know whether eating carrots daily improves a person's eyesight.

A school wants to test the effectiveness of an online program designed to teach writing skills. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.

a. Forty students volunteer to use the online program. Forty other students volunteer to refrain from using the online program. After 6 months, each student is evaluated and it is determined that the students who have been using the online program improved their writing skills more than the other group.

b. The school randomly selects 100 students from each grade. Within each grade, the students are randomly assigned to use the online program or to refrain from using the online program. After 6 months, a significantly large number of the students who used the online program show significant gains in their writing skills.

493 #1, 2, 3, 5, 6, 7, 9, 11, 12, 13, 16, 18, 20, 21, 23 = 15

Algebra 2

9-05 Making Inferences from Sample Surveys

Descriptive Statistics

- _____, _____, _____ of data

Inferential Statistics

- Using a sample to draw _____
- Make _____ (inferences) about the population

The numbers of coupons purchased in the past year by a random sample of 40 adult users of a restaurant discount service are shown in the table. Estimate the population mean μ .

Number of Coupons				
45	32	44	49	33
31	36	55	51	38
66	40	71	9	27
104	14	18	11	64
22	3	38	50	18
28	12	33	44	21
41	19	35	25	39
49	27	45	24	41

Church leaders want to know if youth like their Sabbath School. They conduct several surveys of randomly selected youth. The results are shown in the table.

Based on the first 2 surveys, do you think more youth like Sabbath School?

Based on all the surveys, do you think more youth like Sabbath School?

Sample Size	Number of "Yes" Responses	Percent of "Yes" Responses
3	2	66.7%
15	11	73.3%
40	16	40%
60	25	41.7%
105	46	43.8%
160	72	45%
200	94	47%

A national polling company claims 28% of U.S. adults say students should be required to participate in a physical education class every school day. You survey a random sample of 50 adults.

a. What can you conclude about the accuracy of the claim when 16 adults in your survey agree?

b. What can you conclude about the accuracy of the claim when 21 adults in your survey agree?

Margin of Error

- In normal distributions, _____ of data is within _____ standard deviations of the _____
- 2 standard deviations \approx _____

If a standard deviation is about 0.06, find and interpret the margin of error.

Margin of Error (95% confidence)

$$Error = \pm \frac{1}{\sqrt{n}}$$

- True result likely between
 - $p - \frac{1}{\sqrt{n}}$ and $p + \frac{1}{\sqrt{n}}$

In a survey of 2680 U.S. adults, 34% said that movies are their main source of entertainment. Give an interval that is likely to contain the exact percent of U.S. adults who think movies are their main source of entertainment.

501 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 33, 35, 41, 43, 45, 47 = 20

9-06 Making Inferences from Experiments

c. What can you conclude from parts (a) and (b)?

Total Yield of Green Bell Pepper Plants (kilograms)	
Control Group	Treatment Group
0.8	1.0
0.5	1.1
0.6	0.9
0.7	0.8
0.9	1.2
1.1	0.8
0.8	0.6
1.0	1.3
0.6	1.1
0.9	0.9

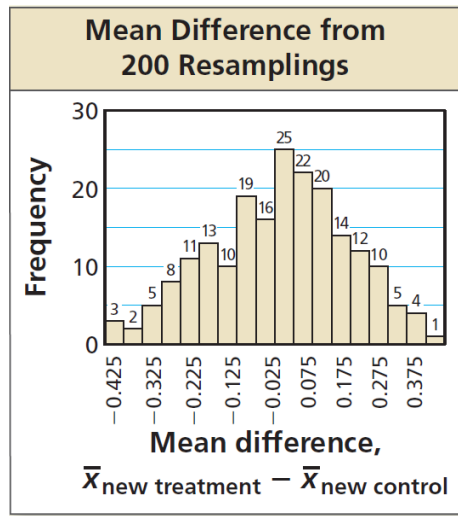
- When the sample size is too _____ to be meaningful

1. Combine _____ the results
2. Assign each data point a _____
3. Use a _____ to select all the data points
4. Make the first half the new _____ and the other half the new _____
5. Find $\bar{x}_{treatment} - \bar{x}_{control}$
6. Repeat _____ times and draw a _____ of the differences
7. If the experimental difference is near the _____ of the histogram, then it is significant

PRB \rightarrow randint(1,20)

Total Yield of Green Bell Pepper Plants (kilograms)	
Control Group	Treatment Group

The histogram shows the results from 200 resamplings of the data. Compare the experimental difference of 0.18 from Example 1 with the resampling differences. What can you conclude about the hypothesis? Does tap water have an effect on the yield?



- If no _____ is given, use the resampling simulator at andrews.edu/~rwright/algebra2/ResamplingSimulator.html

509 #1-10 all, 13-16 all, 20 = 15

Algebra 2

9-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

9-01

A normal distribution has a mean of 150 and a standard deviation of 10. Find the probability that a randomly selected x -value from the distribution is in the given interval.

1. Less than 130
2. Between 140 and 170
3. A study found that the number of students per classroom is normally distributed with a mean of 18 students and a standard deviation of 3 students. What is the probability that a randomly selected classroom will have less than 21 students?

9-02

Does the situation describe a parameter or statistic?

4. A school surveys all the students about whether they want cookies every day for lunch.
5. In a survey of 500 farmers, 76% say they like living in the country.

Identify the population and sample.

6. A restaurant manager asks 15 of the people in their dining room whether they enjoy the music.
7. A scientist is studying monarch butterflies by catching 25 and measuring their wings.

9-03

Identify the type of sample described. Then tell if the sample is more likely to be biased or unbiased.

8. A student is surveying people in her town to find out how involved they are in their local charities. The student asks only his or her own family members.
9. A faculty committee wants to know what the students think about taking extra math classes. They have a computer to generate a list of 20 students to survey.

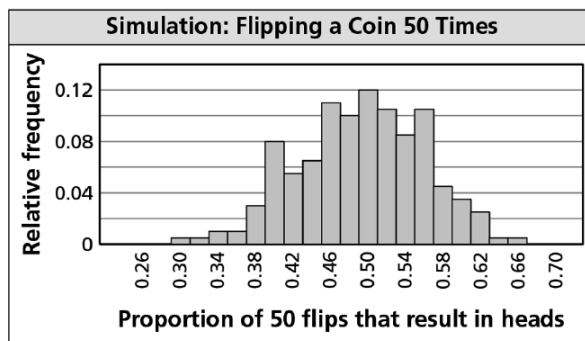
9-04

Determine whether the study is a randomized comparative experiment.

10. A recent study shows that adults who rise before 6:30 a.m. are better drivers than other adults. The study monitored the driving records of 140 drivers who always wake up before 6:30 a.m. and 140 drivers who never wake up before 6:30 a.m. The early risers had 12% fewer accidents.
11. A scientist studies the effects of a certain pollen on mice. He takes 30 mice and randomly assigns them into two groups. One group is exposed to pollen and the other groups is not.

Determine whether the following research topic is best investigated through an experiment or an observational study.

12. You want to know if weeding a flower bed every day leads to better quality flowers.
13. You want to know which yard in the neighborhood has more mosquitos.
14. You flip a coin 4 times and do not get a tails. You suspect this coin favors heads. The coin maker claims that the coin does not favor heads or tails. You use technology to simulate 200 random samples of flipping a coin 50 times. The histogram shows the results. What should you conclude when you flip the actual coin 50 times and get 25 heads?



9-05

Find the margin of error for a survey with the given sample size. Round your answers to the nearest tenth of a percent.

15. 5000

16. 100

Find the sample size required to achieve the given margin of error. Round your answers to the nearest whole number.

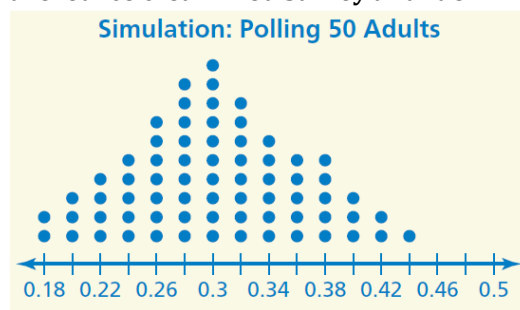
17. $\pm 2\%$ 18. $\pm 20\%$

19. In a survey of 312 people at the local shopping center, 73% favored eating food. Find the margin of error for the survey, and give an interval that is likely to contain the exact percent of all people who favor eating food.

A national polling company claims that 30% of U.S. adults like guacamole flavored ice cream. You survey a random sample of 50 adults. Use the results of the simulation.

20. What can you conclude about the accuracy of the claim that the population proportion is 0.30 when 21 adults in your survey like guacamole flavored ice cream?

21. What can you conclude about the accuracy of the claim that the population proportion is 0.30 when 15 adults in your survey like guacamole flavored ice cream?

**9-06**

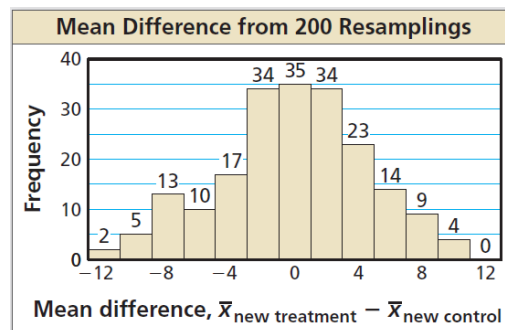
A randomized comparative experiment tests whether a supplement affects resting heart rate.

	Heart Rate (beats per minute)									
Control Group	88	72	72	63	66	74	81	76	71	74
Treatment Group	74	81	83	81	84	76	90	82	81	83

22. Find the experimental difference of the mean yield of the treatment group, $\bar{x}_{treatment}$, and the mean yield of the control group, $\bar{x}_{control}$.

A randomized comparative experiment tests whether a premium oil affects the lifespan of a particular engine. The control group involved six engines maintained using generic oil and the treatment group involved six engines maintained using premium oil. Analyze the hypothesis below. The histogram shows the results from 200 resamplings of the data.

The premium oil has no effect on lifespan.



23. Compare the experimental difference, $\bar{x}_{treatment} - \bar{x}_{control} = 9 \text{ months}$, with the resampling differences. What can you conclude about the hypothesis? Does the premium oil increase the lifespan of the engine?

Answers

- | | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>1. 2.5%</p> <p>2. 81.5%</p> <p>3. 84%</p> <p>4. Parameter</p> <p>5. Statistic</p> <p>6. Population: all the people in the dining room; Sample: the 15 people surveyed</p> <p>7. Population: all monarch butterflies; Sample: the 25 that were caught and measured</p> <p>8. Convenience; biased</p> <p>9. Random; unbiased</p> | <p>10. Not a randomized comparative experiment because it is not random. There may be other reasons such as maybe the early risers go to bed earlier and get more sleep than the late risers.</p> <p>11. Yes</p> <p>12. Experiment</p> <p>13. Observation</p> <p>14. Does not favor head or tails</p> <p>15. $\pm 1.4\%$</p> <p>16. $\pm 10\%$</p> <p>17. 2500</p> <p>18. 25</p> | <p>19. $Error = \pm 5.7\%$, 67.3% to 78.7%</p> <p>20. Not accurate</p> <p>21. Accurate</p> <p>22. 7.8 beats per minute</p> <p>23. 9 is at the tail of the histogram. Thus, premium oil has an effect on the engine.</p> |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|